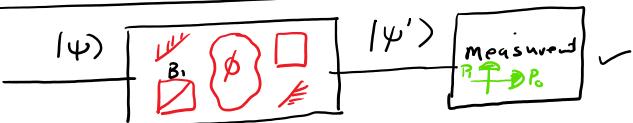


QC Lecture 3.

1. Notation, ket $|\psi\rangle$, quantum states, superpositions

2. Quantum interferometer, phase ϕ
Measurement, inner products



Initialization Create a superposition
✓ Preserve
✓ Q. Process fast enough

Decoherence

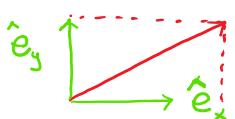
Quantum Logic gates
Circuit models

Basis qubit

vector 2D space

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle \quad \text{general}$$

$$|c_0|^2 + |c_1|^2 = 1$$



$$\vec{a} = a_x \hat{e}_x + a_y \hat{e}_y$$

basis vectors $\{\hat{e}_x, \hat{e}_y\}$

Basis kets

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2-Dim

$$\begin{aligned} \text{Qubit } |\psi\rangle &= c_0|0\rangle + c_1|1\rangle \\ &= \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} \leftarrow \text{Representation} \end{aligned}$$

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}} e^{i\pi/6}|1\rangle = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} e^{i\pi/6} \end{pmatrix}$$

Every ket \iff column vector

$$\overline{\langle \phi | \psi \rangle} = f(\langle \phi |, |\psi \rangle) = \underbrace{\langle \phi |}_{\text{view}} \underbrace{\downarrow}_{\text{bra}} \langle \phi | \psi \rangle$$

Inner product number

$$|\phi\rangle = \begin{pmatrix} c_0 \\ c_1 \end{pmatrix}$$

$$|\psi\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$\langle \phi | = \begin{pmatrix} c_0^* \\ c_1^* \end{pmatrix}$$

$$\overbrace{\quad \quad}^{\text{number}}$$

....

$$\langle \phi | \psi \rangle = \overbrace{c_0^* c_1^*}^{\text{number}} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \underbrace{c_0^* a_0 + c_1^* a_1}_{\text{number}}$$

$$\underbrace{\langle \psi | \psi \rangle}_{\text{itself}} = \underbrace{\begin{pmatrix} a_0^* & a_1^* \\ a_0 & a_1 \end{pmatrix}}_{P_1} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = a_0^* a_0 + a_1^* a_1 = |a_0|^2 + |a_1|^2 = 1$$

Single qubit

$$\begin{aligned} |0\rangle &\xrightarrow{H} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ |1\rangle &\xrightarrow{H} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{aligned}$$

A device implements Q. logic gate.

Q. states \iff kets \iff Col. vectors

Look at this

Q. operations or gates \iff Matrices

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Let's verify

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$B = \underset{2 \times 2}{\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}}$$

gates = \bigcup_H unitary

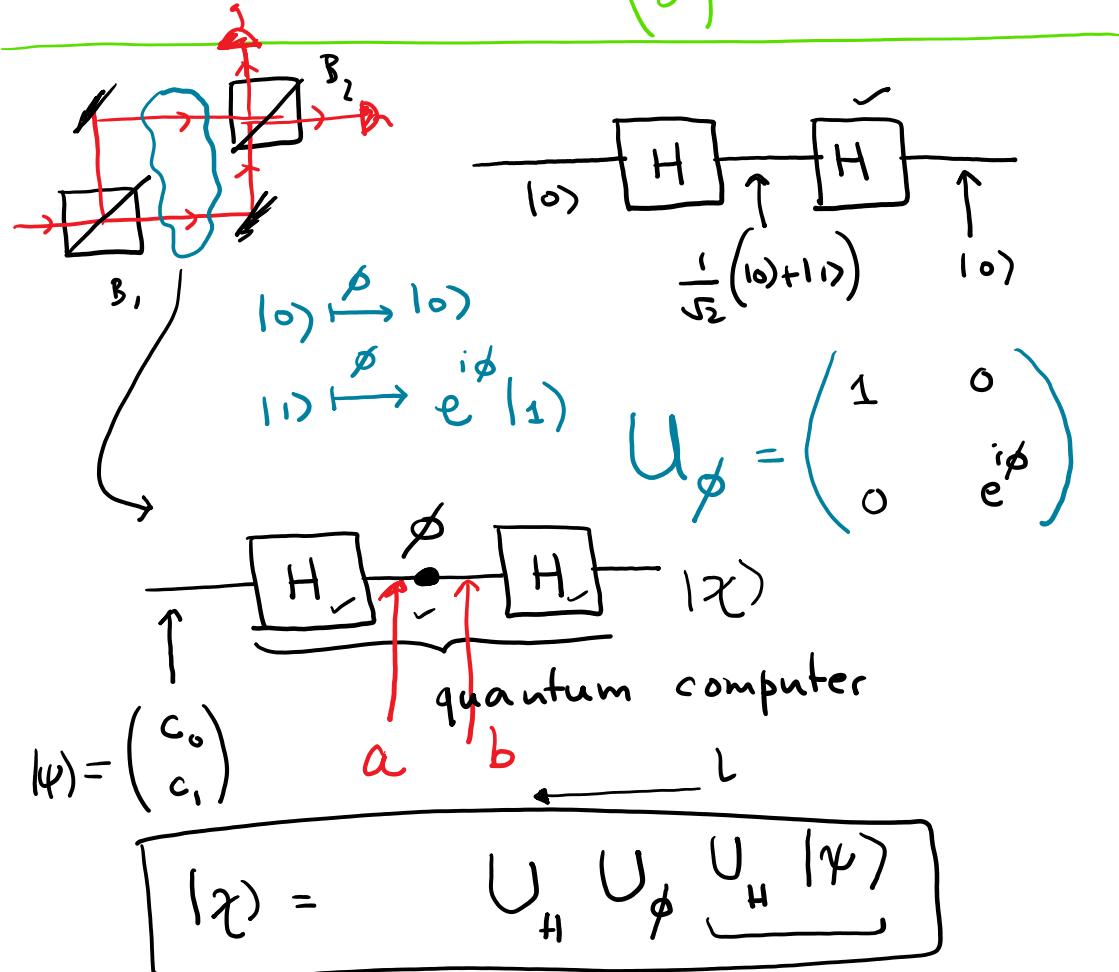
$B|0\rangle$ $B|1\rangle$

$$\left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right) = 0$$

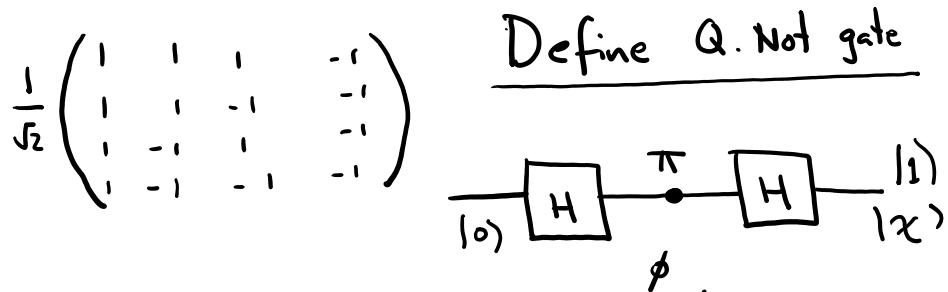
$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

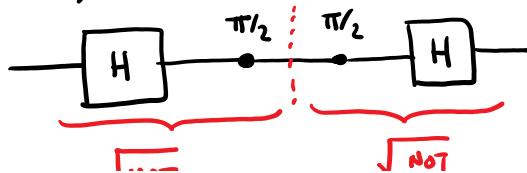
$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$



$$|x\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix}$$



$$\begin{aligned} |x\rangle &= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 \\ 1+1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle \end{aligned}$$



$U_{\text{NOT}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
 $|0\rangle \mapsto |1\rangle$
 $|1\rangle \mapsto |0\rangle$

Single qubit gates (at least)
Unitary operations

$$U |\psi\rangle = |\phi\rangle \quad \text{preserves the norm}$$

If $\langle \psi | \psi \rangle = 1 \Rightarrow \langle \phi | \phi \rangle = 1$

Bloch Sphere

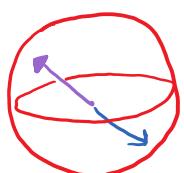
Are they same global phase

$$\frac{|1\rangle - i|0\rangle}{\sqrt{2}} = \frac{-i(|0\rangle + |1\rangle)}{\sqrt{2}}$$

$$= e^{-i\pi/2} \left(\frac{|0\rangle + i|1\rangle}{\sqrt{2}} \right) = |\chi\rangle$$

$$\langle 0 | \chi \rangle = \frac{e^{-i\pi/2}}{\sqrt{2}}$$

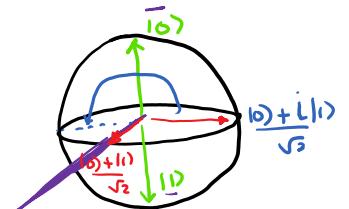
$$|\langle 0 | \chi \rangle|^2 = \frac{1}{2} e^{i\pi/2} e^{-i\pi/2} = \frac{1}{2}$$



$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \xrightarrow[U_{\text{NOT}}]{\text{NOT}} \frac{|1\rangle + |0\rangle}{\sqrt{2}} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Rotation on a Bloch sphere

① axis of rotation



② Amount of rotation

NOT = rotation about the $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$ axis = $|X\rangle$ of the Bloch sphere by π .

Hadamard gate $U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$e^{-i\pi \hat{\sigma}_x/2}$$