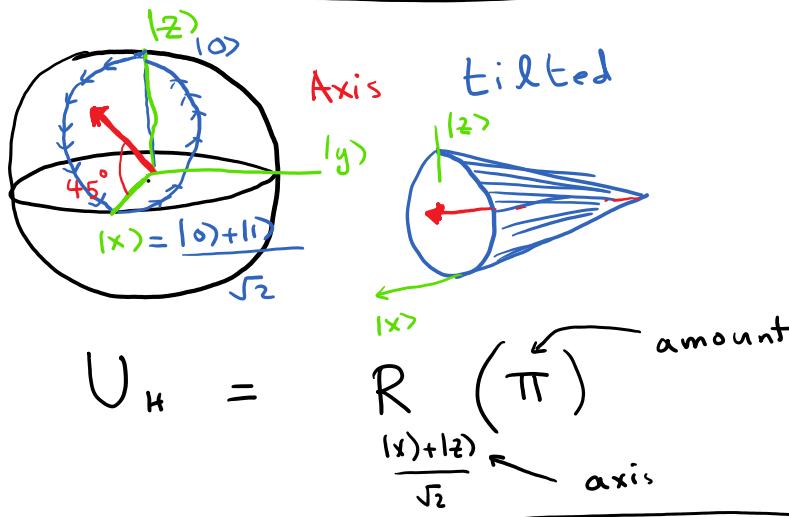
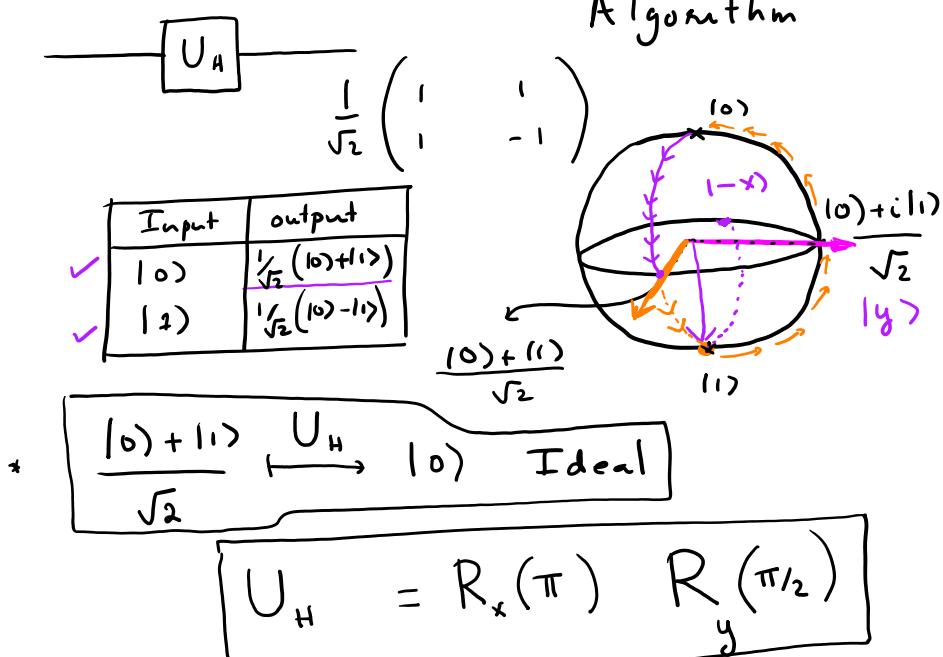


Lecture 4 Quantum Algorithm



Quantum circuit diagram showing two qubits. The first qubit passes through a unitary U , and the second qubit passes through a sequence of unitaries U_1 and U_2 .

$|0\rangle \otimes |0\rangle = |0\rangle |0\rangle = |00\rangle$

$|0\rangle \otimes |0\rangle$ is the tensor product of the 1st and 2nd qubits.

$|\psi\rangle = C_{00}|00\rangle + C_{01}|01\rangle + C_{10}|10\rangle + C_{11}|11\rangle$

$|C_{00}|^2 + |C_{01}|^2 + |C_{10}|^2 + |C_{11}|^2 = 1$

$|\psi\rangle^n$ is an n -qubit state:

$$|\psi\rangle^n = \sum_i^n C_i |i\rangle$$

$|\psi\rangle = |0\rangle \otimes |1\rangle$

$|\psi\rangle$

$|01\rangle \xrightarrow{\text{matrix notab}} |01\rangle = |0\rangle \otimes |1\rangle$

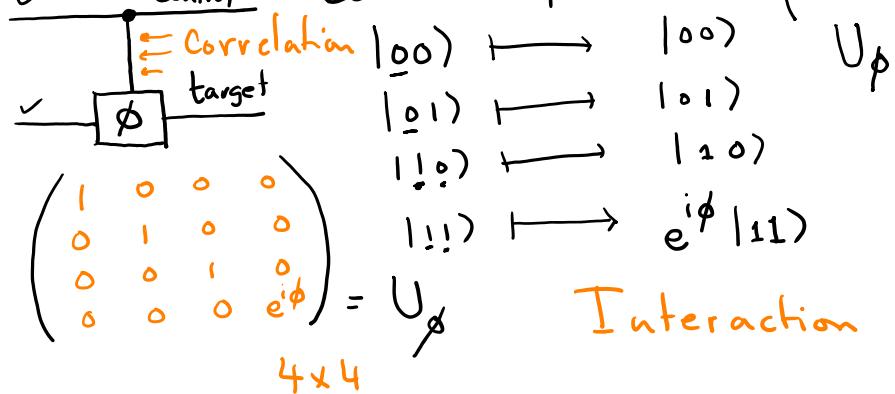
$$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

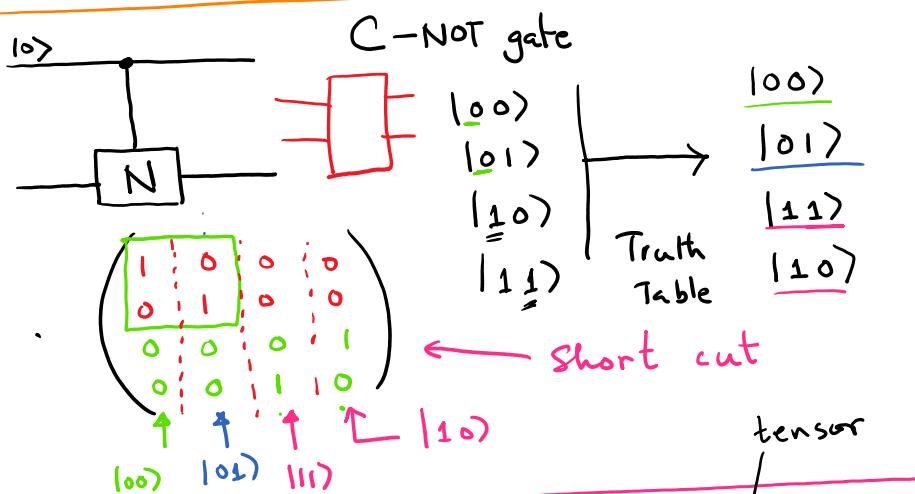
$$|\psi\rangle = \begin{pmatrix} C_{00} \\ C_{01} \\ C_{10} \\ C_{11} \end{pmatrix}$$

2 qubit quantum gate An example

✓ control Controlled phase gate (Def.)



C- π $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Controlled π gates



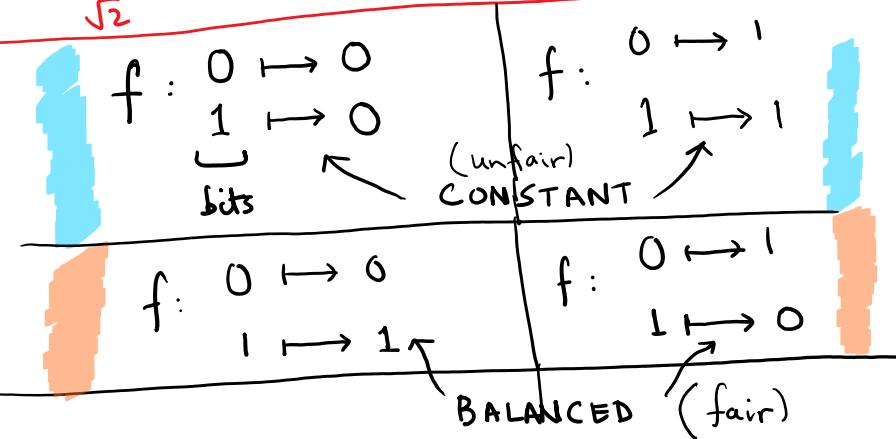
Q. Parallelism
Deutsch algorithm

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$$

$i\phi_{1..n}$

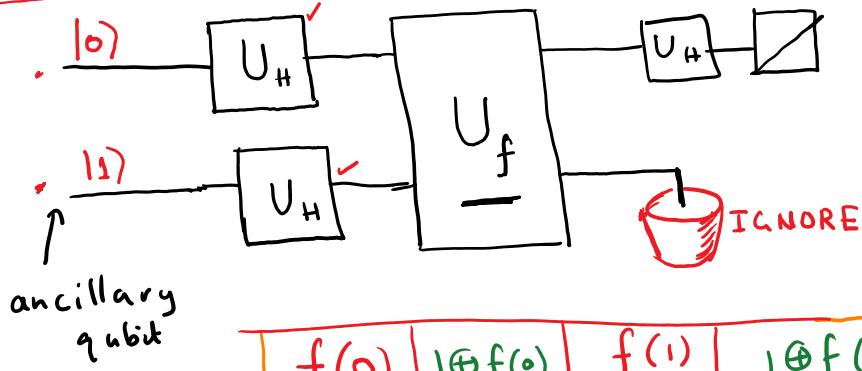
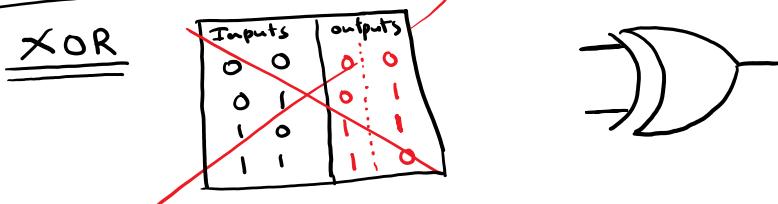
$$|0\rangle \otimes \frac{1}{\sqrt{2}} (|0\rangle + e^{i\phi} |1\rangle) = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} e^{i\phi} |01\rangle$$

$$\begin{pmatrix} ad \\ bc \\ bd \end{pmatrix}$$



$$\frac{f(0)}{f(1)} = ?$$

Two evaluations classically



At the back of my mind

$f(0)$	$1 \oplus f(0)$	$f(1)$	$1 \oplus f(1)$
0	1	0	1
1	0	1	0

$$0 \oplus \boxed{\text{anything}} = \boxed{\text{anything}}$$

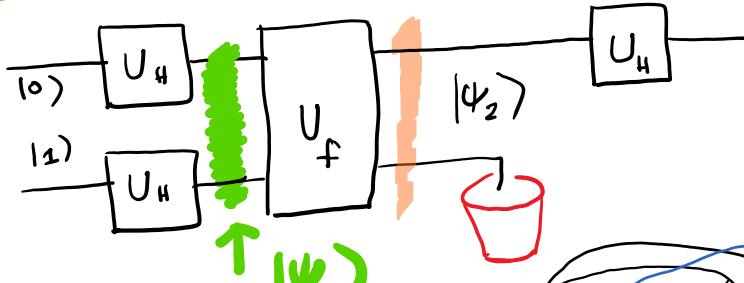
U_f

a	b	$a \oplus b$	XOR
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

$$\begin{aligned}
 |00\rangle &\mapsto |0\rangle |f(0)\rangle & \checkmark \\
 |01\rangle &\mapsto |0\rangle |1 \oplus f(0)\rangle & \checkmark \\
 |10\rangle &\mapsto |1\rangle |f(1)\rangle \\
 |11\rangle &\mapsto |1\rangle |1 \oplus f(1)\rangle
 \end{aligned}$$

L11 TQ

$$|x\rangle|y\rangle \xrightarrow[\text{general}]{U_f} |x\rangle|y\oplus f(x)\rangle \xrightarrow{\text{preserved}} |1\rangle|i\oplus f(1)\rangle$$

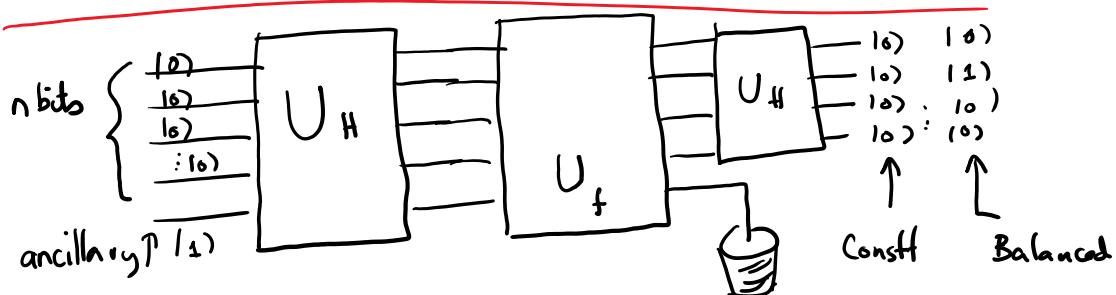
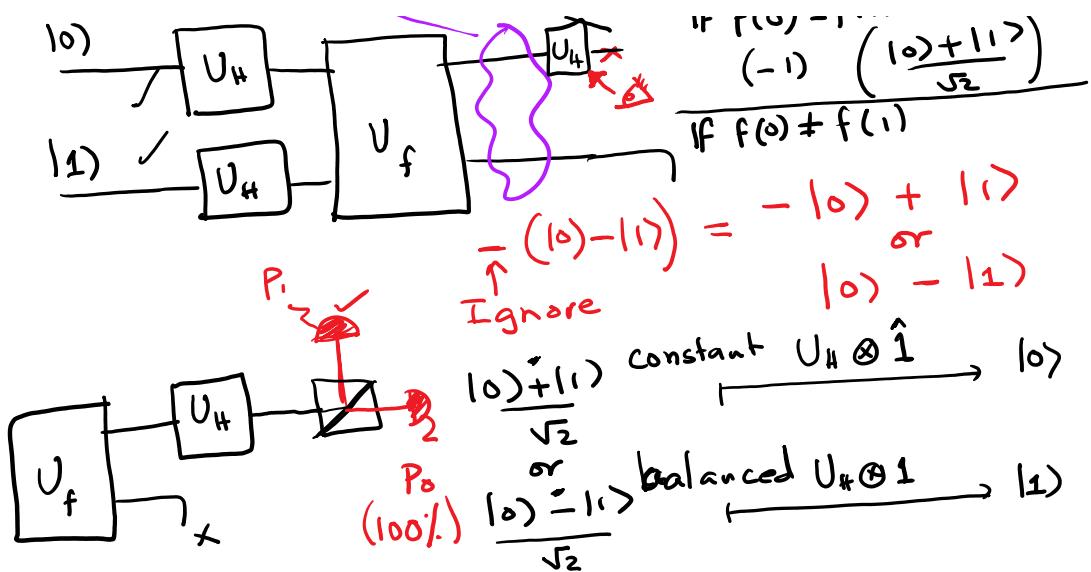


$$\begin{aligned}
 & |0\rangle|1\rangle \xrightarrow{U_H \otimes U_H} |\Psi_1\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \\
 &= \frac{1}{2} \left(|00\rangle - |01\rangle + |10\rangle - |11\rangle \right) \\
 &\xrightarrow{U_f} \frac{1}{2} \left(|0\rangle|f(0)\rangle - |0\rangle|i\oplus f(0)\rangle + |1\rangle|f(1)\rangle - |1\rangle|i\oplus f(1)\rangle \right) \\
 &\quad \text{NOTICE} \\
 &\quad \underbrace{(-1)^{f(0)} |0\rangle}_{\text{f}(0)} \otimes (|0\rangle - |1\rangle) \quad \underbrace{(-1)^{f(1)} |1\rangle}_{\text{f}(1)} \otimes (|0\rangle - |1\rangle) \\
 &\quad \stackrel{\perp}{=} |0\rangle|0\rangle - |0\rangle|1\rangle \quad \text{Home work} \\
 &\quad = |0\rangle \otimes (|0\rangle - |1\rangle)
 \end{aligned}$$

$$\begin{aligned}
 & |0\rangle|1\rangle - |0\rangle|0\rangle \\
 &= |0\rangle(|1\rangle - |0\rangle) = (-1)|0\rangle \otimes (|0\rangle - |1\rangle) \\
 &|\Psi_2\rangle = \frac{1}{\sqrt{2}} \left((-1)|0\rangle \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) + (-1)|1\rangle \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 |\Psi_2\rangle &= \left(\frac{(-1)^{\overline{f}(0)} |0\rangle + (-1)^{\overline{f}(1)} |1\rangle}{\sqrt{2}} \right) \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \\
 &\quad \text{1st qubit} \quad \text{2nd qubit} \\
 &\quad \text{if } f(0) = f(1) \text{ CONSTANT} \\
 &\quad (-1) \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right)
 \end{aligned}$$





2^n { $000 \dots 0$ $n-1$ expts
 $000 \dots 1$
 $010 \dots$

$f: \{0, 1\}^n \rightarrow \begin{cases} 0 & \text{always} \\ 1 & \text{always} \end{cases}$ const
 sometimes 0, sometimes 1