Lectur 1: Practical Quantum Computing

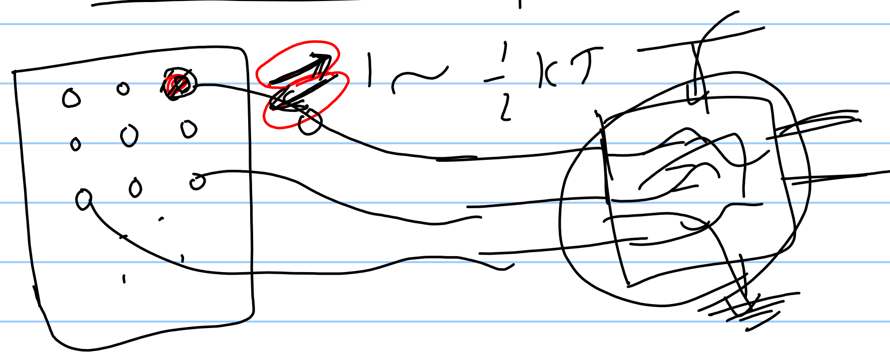
Note Title 24/03/2021

- Intro — qubits, quantum computers, entanglement, Teleportation
- Circuit model, gates
- Simple but power Algorithm
  - Deutsch – Jozsa
  - B – V
- Grover's search
- QFT
  - Factoring

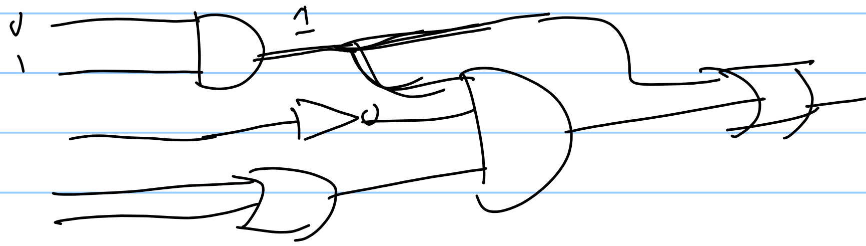
Today:

- Classical vs. Quantum computers
- Qubit, notation
- Jupyter

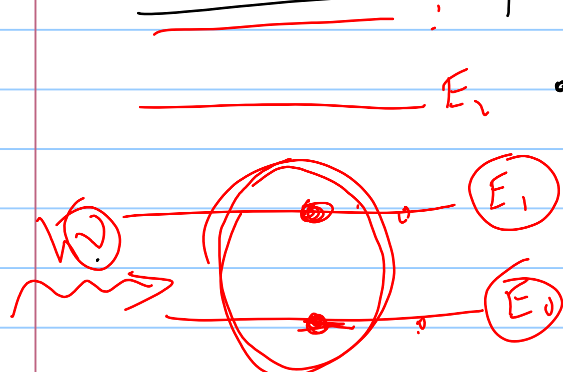
Classical Computers:



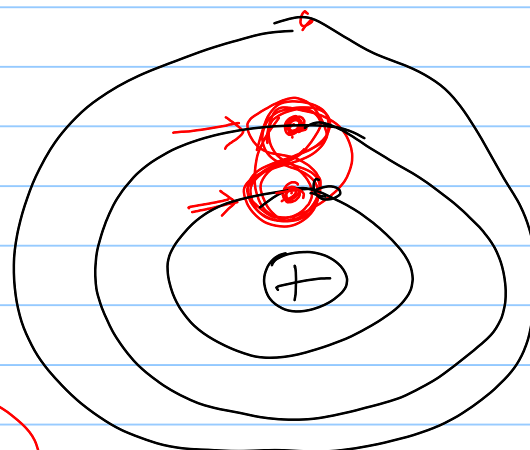
0 — 0.5 V → 0
1 — 5 V → 1



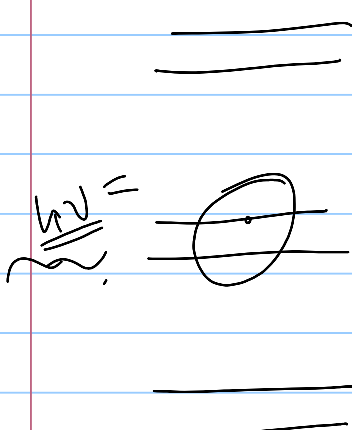
Quantum Computer:



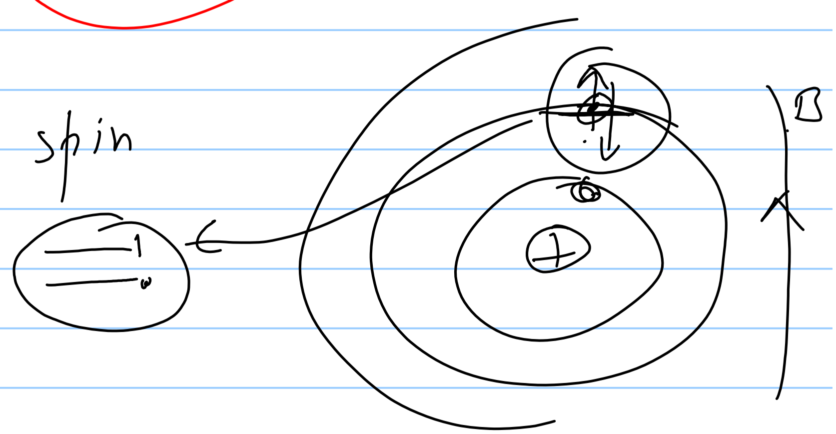
• Hydrogen



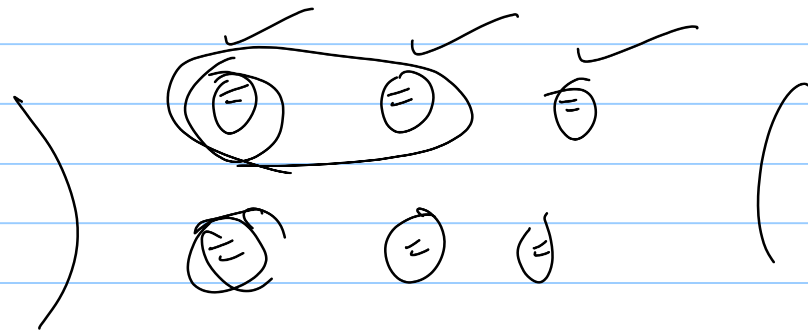
Qubit



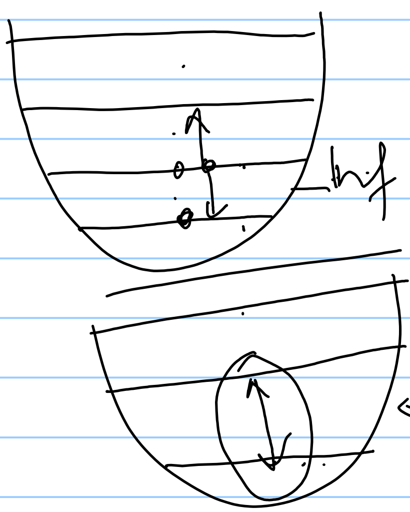
• spin

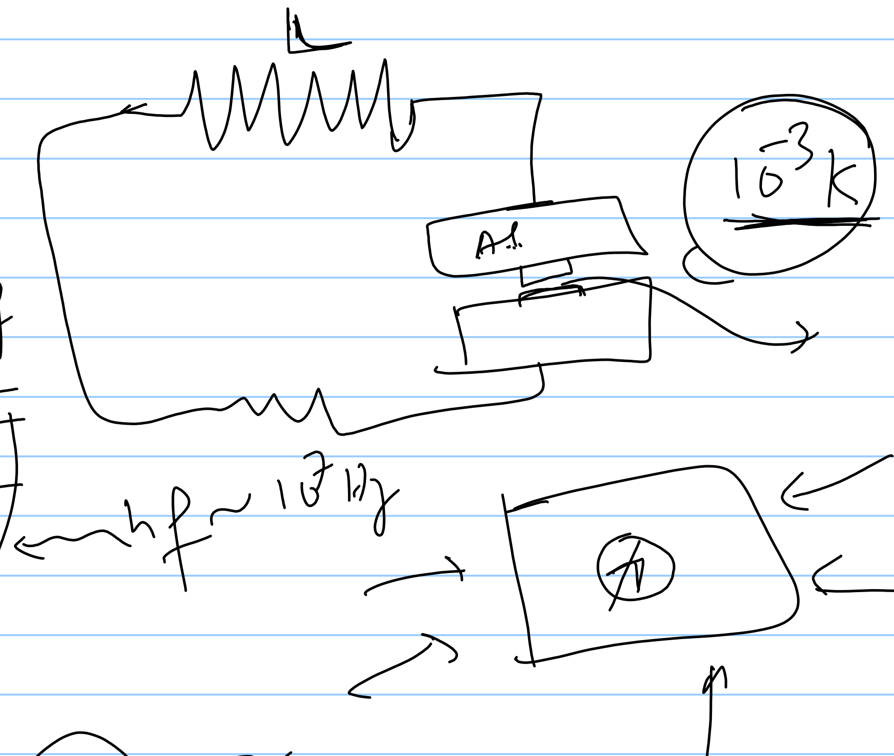


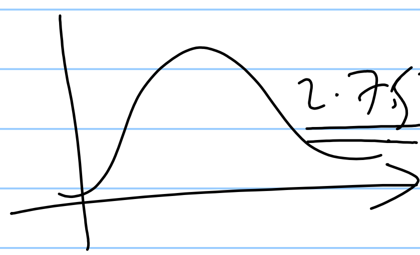
Trapped Ion



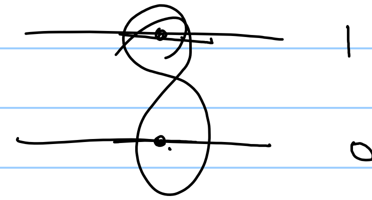
LC



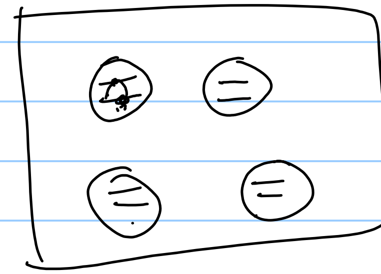


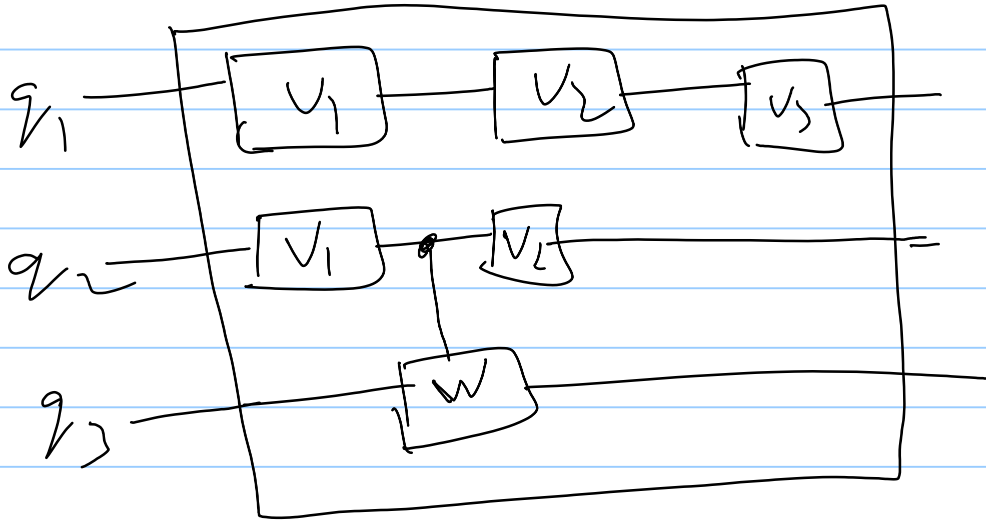


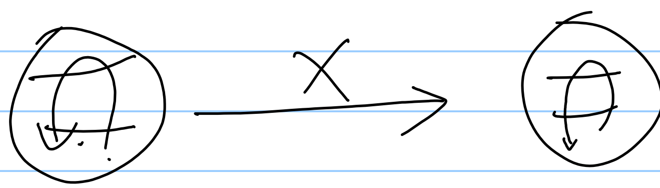
- Summary



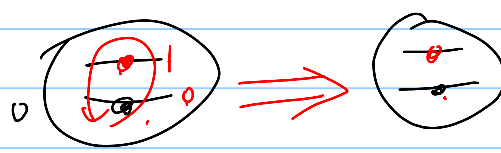
Logical operations

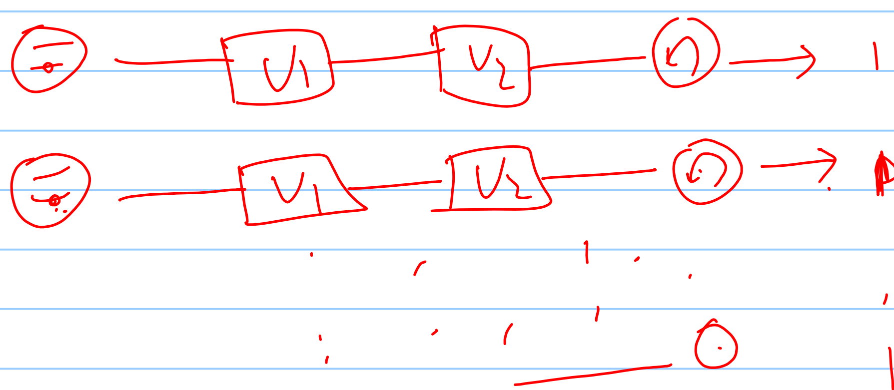




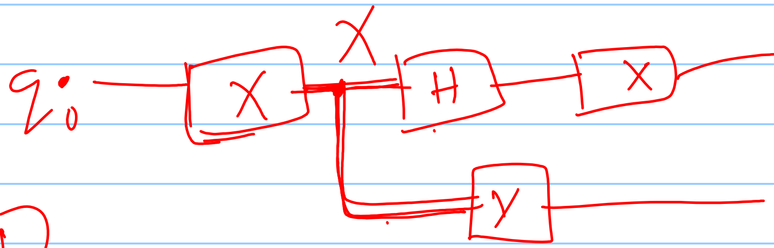


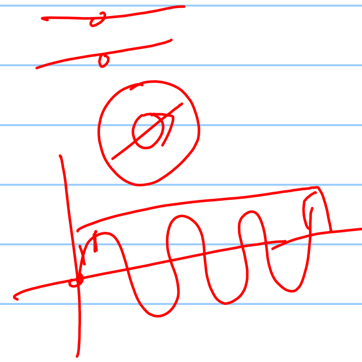
Measurements



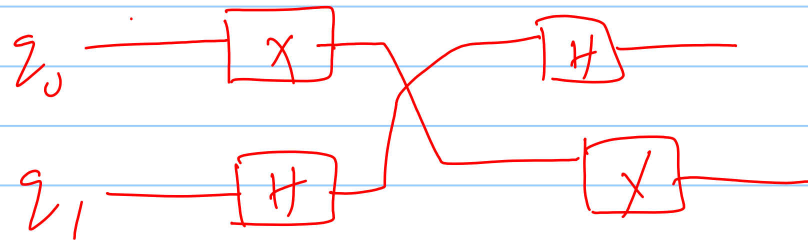


Circuit Model:





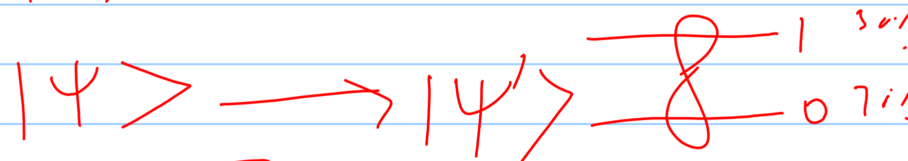
1



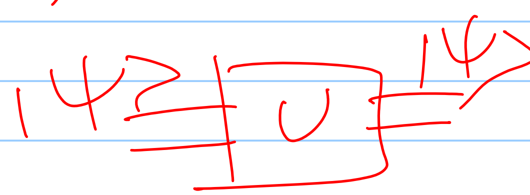
2 All operations are Unitary:

$|\psi\rangle$

Dirac



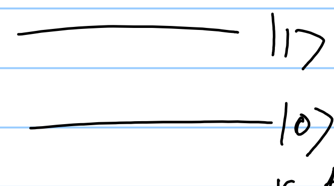
$$\boxed{|\psi'\rangle = U|\psi\rangle}$$



Dirac Notation:

$$|\psi\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

such that $\boxed{|a_1|^2 + |a_2|^2 = 1}$



$$\| |\psi\rangle \| = a_1^2 + a_2^2$$

we define $\langle\psi| = |\psi\rangle^\dagger = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}^\dagger$

$$= \left(a_1^* \quad a_2^* \right)$$

$$\| |\psi\rangle \| = \langle\psi|\psi\rangle = \left(a_1^* \quad a_2^* \right) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$= a_1^* a_1 + a_2^* a_2 = |a_1|^2 + |a_2|^2$$

$$= 1$$

$$|a\rangle = \begin{pmatrix} 3 \\ 4i \end{pmatrix}$$

$i^2 = -1$

$\boxed{i = \sqrt{-1}}$

$$\langle a| = \left(3 \quad -4i \right)$$

Inner product

$$\langle a|a\rangle = \left(3 \quad -4i \right) \begin{pmatrix} 3 \\ 4i \end{pmatrix} = 9 + 16 = 25$$

$$|a\rangle = \begin{pmatrix} 3/5 \\ \frac{4}{5} i \end{pmatrix}$$

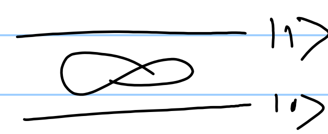
Basis:

$$|\psi\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = a_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

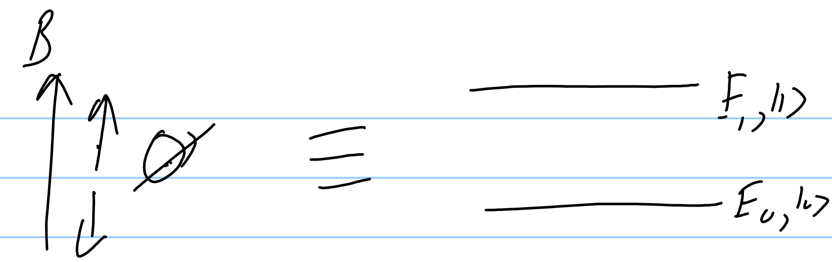
$$\boxed{|\psi\rangle = a_1 |0\rangle + a_2 |1\rangle}$$

$$\{ |0\rangle, \; |1\rangle \}$$ Computational Basis

$$p(0) = |a_1|^2 = a_1 a_1^*$$



$$p(1) = |a_2|^2 = a_2 a_2^*$$



$$\Psi = \frac{3}{5}|0\rangle + \frac{4i}{5}|1\rangle$$

$$\Downarrow$$

$$= b_1|0\rangle + b_2|1\rangle$$

Bans

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$\langle+|+\rangle = \left[\frac{1}{\sqrt{2}}\left(\langle0| + \langle1|\right)\right]\left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\right)$$

$$= \frac{1}{2}\left[\langle0|0\rangle + \langle1|0\rangle + \langle1|0\rangle + \langle1|1\rangle\right]$$

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \Longrightarrow \langle0|0\rangle = (1 \quad 0)\begin{pmatrix}1\\0\end{pmatrix} = 1$$

$$\langle1|0\rangle = (0 \quad 1)\begin{pmatrix}1\\0\end{pmatrix} = 0$$

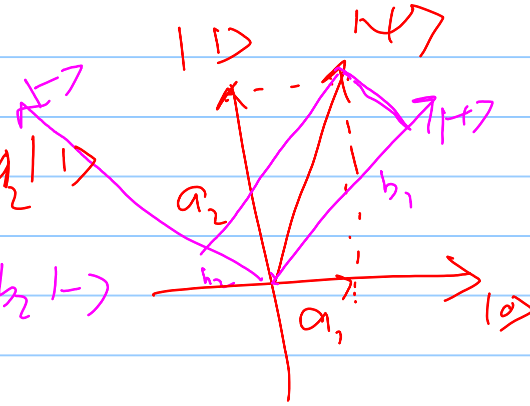
$$\langle+|+\rangle = \frac{1}{2}\left(1 + 0 + 0 + 1\right) = 1$$

Visulize:

If our were real:

$$|\psi\rangle = a_1|0\rangle + a_2|1\rangle$$

$$= b_1|+\rangle + b_2|-\rangle$$



$$a_1 = a_1' + i a_1'' \quad , \quad a_2 = a_2' + i a_2''$$

Four $\boxed{|a_1|^2 + |a_2|^2 = 1}$ Constant

$$e^{ix} = \cos x + i \sin x$$

3 parameters

$$a_1 = r_1 e^{i\alpha_1} \quad , \quad a_2 = r_2 e^{i\alpha_2}$$

$$|a_1|^2 + |a_2|^2 = 1 \longrightarrow \boxed{r_1^2 + r_2^2 = 1} \Rightarrow r_1^2 = 1 - r_2^2$$

$r_1 \in (0, 1)$

$r_2 \in (0, 1)$

$$\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} = 1$$

$$\frac{\theta}{2} = 0 \rightarrow 90^\circ$$

$$\boxed{\theta = 0 \rightarrow 180^\circ}$$

$$|\psi\rangle = a_1|0\rangle + a_2|1\rangle$$

$$= r_1 e^{i\alpha_1}|0\rangle + r_2 e^{i\alpha_2}|1\rangle$$

$$= e^{i\alpha_1}\left[r_1|0\rangle + r_2 e^{i(\alpha_2 - \alpha_1)}|1\rangle \right]$$

$$|\psi\rangle = e^{i\alpha_1}\left[\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle\right]$$

$$\boxed{\phi = \alpha_2 - \alpha_1}$$

$$\langle 0|\psi\rangle = \langle 0|\left[|\psi\rangle\right]$$

$$= e^{i\alpha_1}\left[\cos\frac{\theta}{2}\langle 0|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\langle 0|1\rangle\right]$$

$$= e^{i\alpha_1}\cos\theta/2$$

$$\boxed{|\langle 0|\psi\rangle|^2 = \cos^2\theta/2}$$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

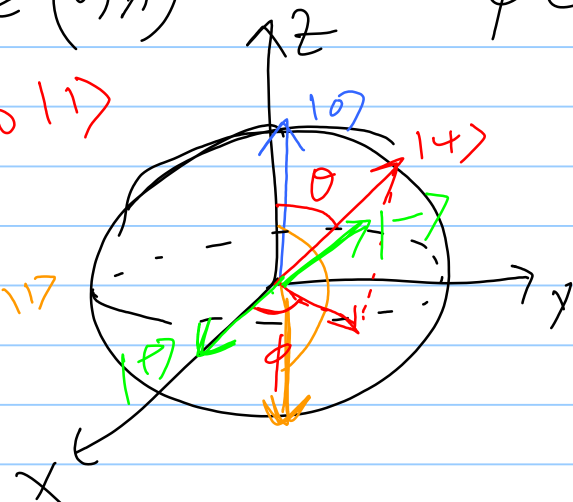
$\theta \in (0, \pi)$ $\qquad$ $\phi \in 0, 360^\circ$

$(0, 2\pi)$

$|0\rangle = 1|0\rangle + 0|1\rangle$

$\boxed{\theta = 0}$

$|1\rangle = 0|0\rangle + 1|1\rangle$



Bloch Sphr

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$\phi = 0$

$\frac{\theta}{2} = 45^\circ$

$\theta = 90^\circ$