

Lecture 2: Gates

Note Title

25/03/2021

$$\underline{|0\rangle\langle 1|} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad \underline{|1\rangle\langle 0|} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\underline{|1\rangle\langle 1|} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\underline{|0\rangle\langle 0|} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\boxed{X = \underline{|0\rangle\langle 1|} + \underline{|1\rangle\langle 0|}}$$

$$\begin{aligned} X|0\rangle &= (\underline{|0\rangle\langle 1|} + \underline{|1\rangle\langle 0|})|0\rangle \\ &= \underline{|0\rangle\langle 1|} \overset{0}{\cancel{|0\rangle}} + \underline{|1\rangle\langle 0|} \overset{1}{\cancel{|0\rangle}} \end{aligned}$$

$$\boxed{X|0\rangle = |1\rangle}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{aligned} |0\rangle &\rightarrow |0\rangle \\ |1\rangle &\rightarrow -|1\rangle \end{aligned}$$

$$\boxed{Z = \underline{|0\rangle\langle 0|} - \underline{|1\rangle\langle 1|}}$$

$$Z|1\rangle = -|1\rangle \quad \leftarrow |1\rangle$$

$$Z|0\rangle = |0\rangle \quad \leftarrow |0\rangle$$

Hadamard

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$Y = i|1\rangle\langle 0| - i|0\rangle\langle 1|$$

$$H = \frac{1}{\sqrt{2}} (|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|)$$

$$H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + 0 + |1\rangle + 0)$$

$$H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle$$



• $X^2 = I$, $Y^2 = I$, $Z^2 = I$
 $H^2 = I$

$$H|+\rangle \rightarrow |0\rangle$$

$$H|-\rangle \rightarrow |1\rangle$$

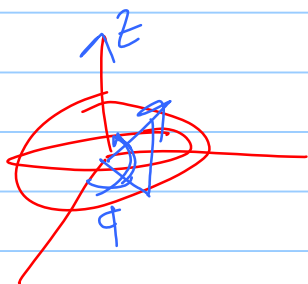
$$\underline{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} X + \frac{1}{\sqrt{2}} Z \\ = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\textcircled{U} = aX + bY + cZ + dI$$

\Rightarrow

Phase gate

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$



$$|0\rangle \rightarrow |0\rangle$$

$$|1\rangle \rightarrow i|1\rangle = e^{i\frac{\pi}{2}}|1\rangle$$

$$\cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle$$

$$\phi \rightarrow \phi + \frac{\pi}{2}$$

$$\Rightarrow S^4 = I$$

$$P(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$