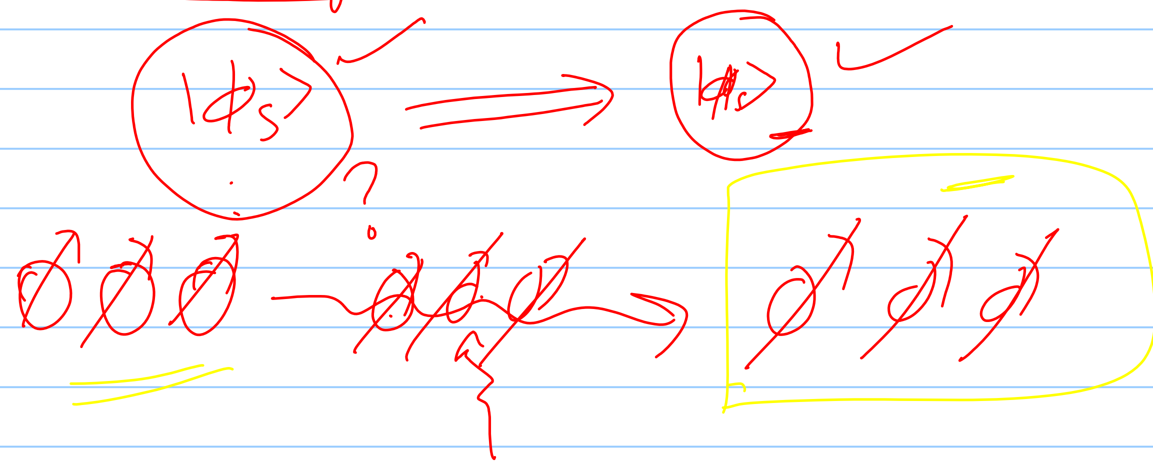
Note Title 21/04/2021

Lecture 9:

- Quantum key distribution / Q. Secure Communication
- Q. State $\xrightarrow[\text{bits}]{\text{classical}}$ Q. state / Teleportation

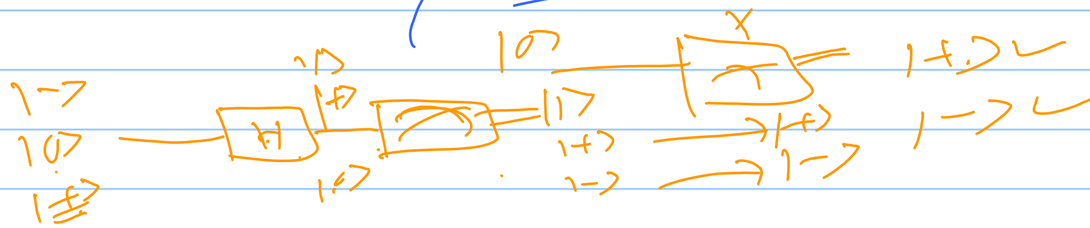
Quantum Key distribution BB84

- No cloning Theorem:

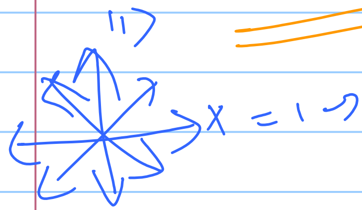


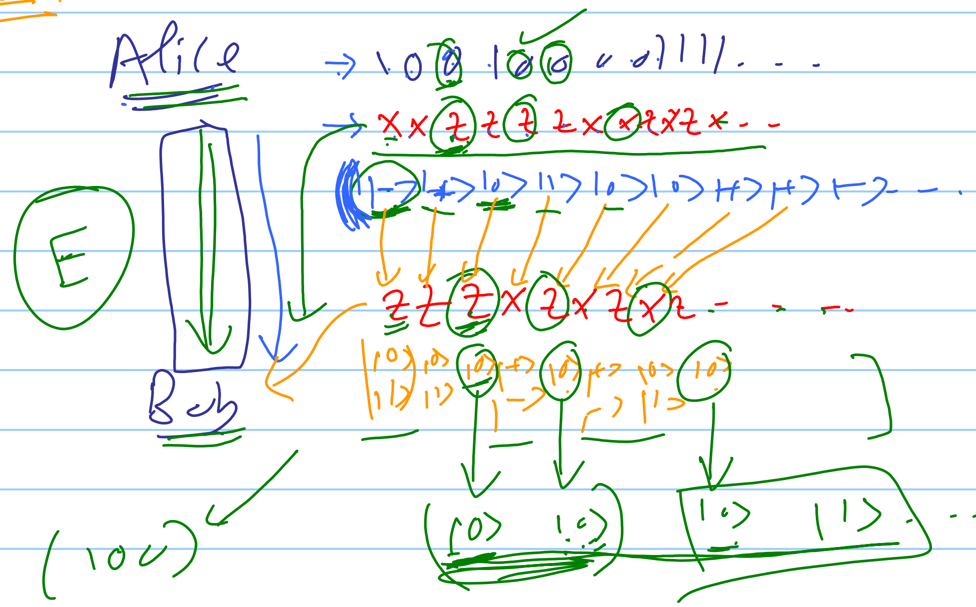
$|\psi\rangle = \begin{cases} |0\rangle \\ |1\rangle \end{cases}$ → Z → $|0\rangle$, $|1\rangle$ Z basis : $|0\rangle$, $|1\rangle$

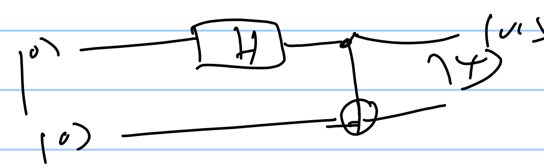
$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$ → Z → $|0\rangle$, $|1\rangle$



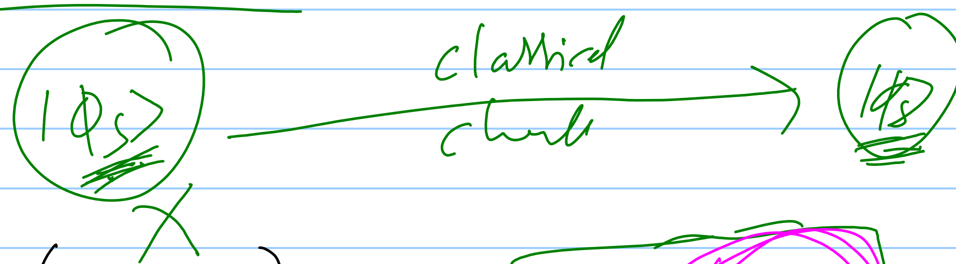
BB84







Teleportation:

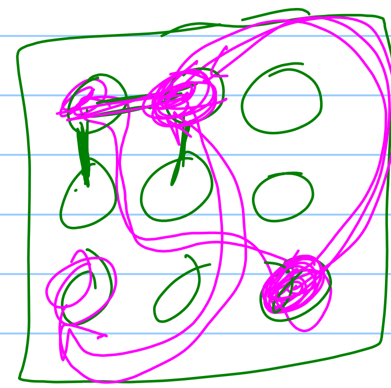


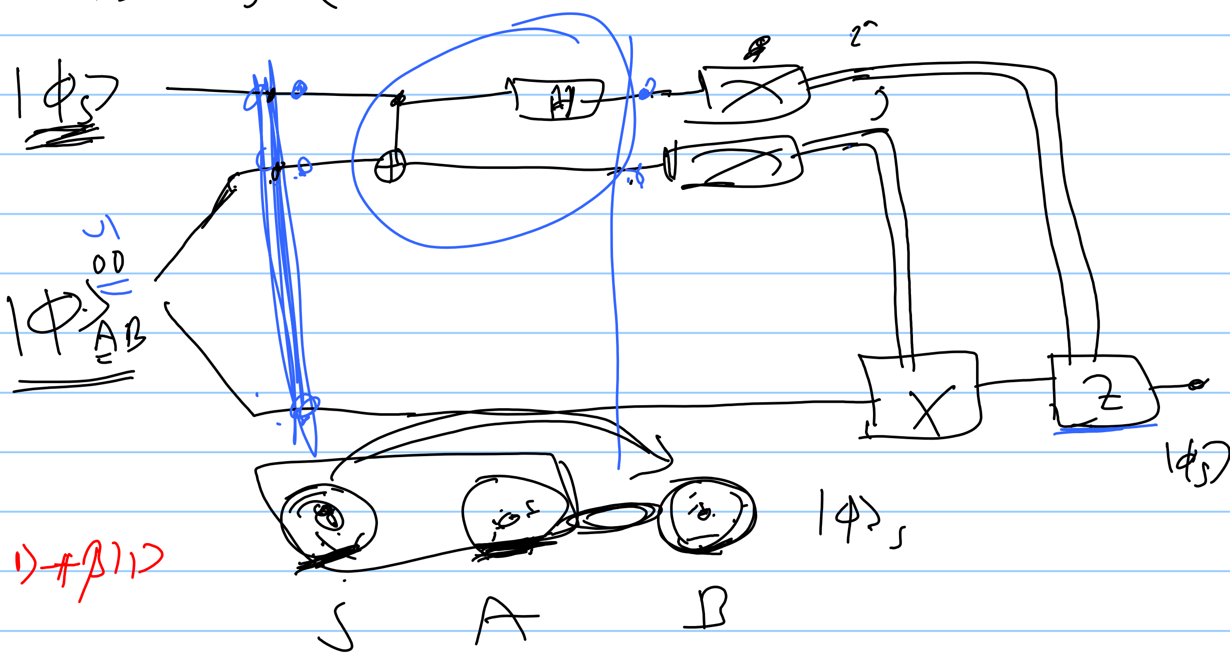
$$|\psi\rangle_{AB}^{00} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\psi\rangle_{AB}^{01} = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

$$|\psi\rangle_{AB}^{10} = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|\psi\rangle_{AB}^{11} = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$





$$|\phi\rangle_s = \alpha|0\rangle + \beta|1\rangle$$

$$|\phi\rangle_s |\psi\rangle^{00}_{AB} = \left(\alpha|0\rangle_s + \beta|1\rangle_s\right)\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle_{SAB} + \alpha|011\rangle_{SAB} + \beta|100\rangle_{SAB} + \beta|111\rangle_{SAB}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

$$= \frac{1}{2\sqrt{2}}\Big[\alpha|000\rangle + \alpha|110\rangle + \alpha|011\rangle + \alpha|101\rangle + \alpha|000\rangle - \alpha|110\rangle + \alpha|011\rangle - \alpha|101\rangle + \beta|100\rangle + \beta|010\rangle + \beta|111\rangle + \beta|001\rangle + \beta|100\rangle - \beta|010\rangle + \beta|111\rangle - \beta|001\rangle\Big]$$

$$= \frac{1}{2\sqrt{2}}\Big[(|00\rangle + |11\rangle)\alpha|0\rangle + (|01\rangle + |10\rangle)\alpha|1\rangle + (|00\rangle - |11\rangle)\alpha|0\rangle + (|01\rangle - |10\rangle)\alpha|1\rangle + (|10\rangle + |01\rangle)\beta|0\rangle + (|11\rangle + |00\rangle)\beta|1\rangle + (|10\rangle - |01\rangle)\beta|0\rangle - (|00\rangle - |11\rangle)\beta|1\rangle\Big]$$

$$= \frac{1}{2}\Big[\left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)(\alpha|0\rangle + \beta|1\rangle) + \left(\frac{|00\rangle - |11\rangle}{\sqrt{2}}\right)(\alpha|0\rangle - \beta|1\rangle) + \frac{(|01\rangle + |10\rangle)}{\sqrt{2}}(\alpha|1\rangle + \beta|0\rangle) + \left(\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right)(\alpha|1\rangle - \beta|0\rangle)\Big]$$

$$= \frac{1}{2}\Big[|\psi\rangle^{00}_{SA}(\alpha|0\rangle + \beta|1\rangle) + |\psi\rangle^{10}_{SA}(\alpha|0\rangle - \beta|1\rangle) + |\psi\rangle^{01}_{SA}(\alpha|1\rangle + \beta|0\rangle) + |\psi\rangle^{11}_{SA}(\alpha|1\rangle - \beta|0\rangle)\Big]$$

$$= \frac{1}{2}\Big[\boxed{|\psi\rangle^{00}_{SA}\,|\phi\rangle_B} + |\psi\rangle^{10}_{SA}\,Z|\phi\rangle_B + |\psi\rangle^{01}_{SA}\,X|\phi\rangle_B + |\psi\rangle^{11}_{SA}\,XZ|\phi\rangle_B\Big]$$

$$Z|\phi_B\rangle \xrightarrow{Z} |\phi\rangle_B$$

$$X|\phi\rangle_B \xrightarrow{X} |\phi\rangle_B$$

$$XZ|\phi\rangle \xrightarrow{ZX}$$

$$ZXXZ = I$$

$$|$$

$$\boxed{HXH = Z}$$

C_Z